CBCS SCHEME

USN | | | | | | | | | | |

17MAT31

Third Semester B.E. Degree Examination, July/August 2021
Engineering Mathematics - III

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions.*

1 a. Obtain the Fourier series for the function

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Hence deduce $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$ **(08 Marks)**

b. Find the Fourier series for the function $f(x) = 2x - x^2$ in $0 < x < 3$. **(06 Marks)**

c. Obtain the constant term and the first sine and cosine terms of the Fourier for y using the following table :

x :	0	1	2	3	4	5
y :	4	8	15	7	6	2

(06 Marks)

2 a. Obtain the Fourier series for the function $f(x) = |\cos x|$, $-\pi < x < \pi$. **(08 Marks)**

b. Find the Half range cosine series for $f(x) = x(\ell - x)$, $0 \le x \le \ell$. **(06 Marks)**

c. Express y as a Fourier series upto first harmonic given :

x :	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y :	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

(06 Marks)

3 a. If $f(x) = \begin{cases} 1 - x^2, & |x| < 0 \\ 0, & |x| \ge 1 \end{cases}$

Find the Fourier transform of f(x) and hence find the value of $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) dx$ **(08 Marks)**

b. Find the Fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{x \sin mx}{1 + x^2} dx \; (m > 0)$ **(06 Marks)**

c. Find $Z_T^{-1}\left[\frac{3z^2 + 2z}{(5z-1)(5z+2)}\right]$. **(06 Marks)**

4 a. Find the Fourier transform of

$$f(x) = \begin{cases} 1 - |x|, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$$ and hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2} dt$. **(08 Marks)**

b. Find the Z – transform of $2n + \sin\left(\frac{n\pi}{4}\right) + 1$. **(06 Marks)**

c. Solve by using Z – transforms $Y_{n+2} - 4\;Y_n = 0$ given that $Y_0 = 0$, $Y_1 = 2$. **(06 Marks)**

5 a. Obtain the lines of regression and hence find the coefficient of correlation for the data :

x :	1	3	4	2	5	8	9	10	13	15
y :	8	6	10	8	12	16	16	10	32	32

(08 Marks)

b. Fit a Second degree parabola in the least Square sense for the following data:

x :	1	2	3	4	5
y :	10	12	13	16	19

(06 Marks)

c. Use the Regula-Falsi method to obtain the real root of the equation $\cos x = 3x - 1$ correct to 3 decimal places in $(0, 1)$. (06 Marks)

6 a. Given the equation of the regression lines $x = 19.13 - 0.87y$, $y = 11.64 - 0.5x$. Compute the mean of x's , mean of y's and the coefficient of correlation. (08 Marks)

b. Fit a curve of the form, $y = a e^{bx}$ for the data:

x :	0	2	4
y :	8.12	10	31.82

(06 Marks)

c. Using Newton-Raphson method to find a real root of $x \log_{10}^{x} = 1.2$ upto 5 decimal places near $x = 2.5$. (06 Marks)

7 a. Given Sin 45° = 0.7071, Sin 50° = 0.7660, Sin 55° = 0.8192, Sin 60° = 0.8660, find Sin 57° using an Backward Interpolation formula. (08 Marks)

b. Applying Lagrange's Interpolation formula inversely find x when y = 6 given the data

x :	20	30	40
y :	2	4.4	7.9

(06 Marks)

c. Using Simpson's $\frac{1}{3}^{rd}$ rule with Seven ordinates to evaluate $\int_2^8 \frac{dx}{\log_{10} x}$. (06 Marks)

8 a. Fit an Interpolating polynomial for the data $u_{10} = 355$, $u_0 = -5$, $u_8 = -21$, $u_1 = -14$, $u_4 = -125$ by using Newton's Divided difference formula and hence find u_2. (08 Marks)

b. Use Lagrange's Interpolation formula to fit a polynomial for the data :

x :	0	1	3	4
y :	-12	0	6	12

Hence estimate y at x = 2. (06 Marks)

c. Evaluate $\int_4^{5.2} \log_e x \, dx$ taking six equal strips by applying Weddle's rule. (06 Marks)

9 a. Using Green's theorem, evaluate $\int_C [(y - \sin x)dx + \cos x \; dy]$, where C is the plane triangle enclosed by the lines $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$. (08 Marks)

b. Using Divergence theorem evaluate $\int_S \vec{F} \cdot ds$, where $\vec{F} = 4x\,i - 2y^2 j + z^2 K$ and S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. (06 Marks)

c. Show that the Geodesics on a plane are straight lines. (06 Marks)

10 a. Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)i - yz^2 j - y^2 z\ K$ over the upper half surface of $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy - plane. **(08 Marks)**

b. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left[\frac{\partial f}{\partial y^1}\right] = 0$. **(06 Marks)**

c. Find the Extremals of the functional

$$\int_{x_0}^{x_1} \frac{y^{1^2}}{x^3} dx.$$ **(06 Marks)**

* * * * *